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Higher-derivative supersymmetric gauge theory

F. S. Gama,[1,*] M. Gomes,[2,†] J. R. Nascimento,[1,‡] A. Yu. Petrov,[1,§] and A. J. da Silva[2,‖]

[1]*Departamento de Física, Universidade Federal da Paraíba, Caixa Postal 5008, 58051-970, João Pessoa, Paraíba, Brazil*

[2]*Departamento de Física Matematica, Instituto de Física, Universidade de São Paulo, Caixa Postal 66318, São Paulo, Brazil*

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We study the one-loop low-energy effective action for the higher-derivative superfield gauge theory coupled to chiral matter.

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I. INTRODUCTION

The use of higher derivatives has been proposed as a way to tame the ultraviolet behavior of physically relevant models. Actually, a finite version of QED was put forward by Lee and Wick about 40 years ago [1]; that proposal was nevertheless beset by the presence of spurious degrees of freedom which induce an indefinite metric in the space of states jeopardizing unitarity, and so required special treatment. Recently, the idea was revived in the so-called Lee-Wick standard model, leading to some new insights into the hierarchy problem [2]. Furthermore, following a similar thread, higher-curvature gravitational models were considered; in spite of the possible breaking of unitarity, they furnish renormalizable quantum gravity models [3] which may be useful for cosmological applications (for a recent review in this direction see [4]).

In supersymmetric models, a higher-derivative regularization method was proposed in [5]. Furthermore, interest in higher-derivative supersymmetric field theories increased not only due to their application within the regularization context (see [6] for many examples) but also for other reasons. For example, a higher-derivative supergravity model, which, in the superconformal sector, can be treated as a natural higher-derivative generalization of the Wess-Zumino model, has been studied in [7,8]. Afterwards, some classical aspects of the very generic class of higher-derivative chiral superfield models have been considered in [9], and the lower perturbative corrections in such theories were obtained in [10].

Therefore, a natural continuation of these studies could be the construction of a consistent higher-derivative gauge theory coupled to chiral matter. In this paper, we construct one such model. We calculate the one-loop low-energy effective action for such a theory, generalizing the results of [11] where the usual super-Yang-Mills field coupled to chiral matter has been studied in the one-loop approximation. In this study, we employ the superfield approach for calculating the supersymmetric effective potential developed in Refs. [12–14]. Moreover, due to the difficulties inherent to the formulation of higher-derivative theories in Minkowski space [15], in the present work we will restrict ourselves to Euclidean space.

II. EFFECTIVE ACTION IN HIGHER-DERIVATIVE SUPERFIELD GAUGE THEORIES: GENERAL APPROACH

It is well known [16] that under the usual gauge transformation the chiral (and antichiral) matter field and the gauge superfield v transform as

$$\phi \to e^{-i\Lambda}\phi, \qquad \bar{\phi} \to \bar{\phi}e^{i\bar{\Lambda}}, \qquad e^{gv} \to e^{-ig\bar{\Lambda}}e^{gv}e^{ig\Lambda}, \tag{1}$$

where Λ is a chiral parameter, and $\bar{\Lambda}$ is an antichiral one. The superfield strength

$$W_\alpha = \bar{D}^2(e^{-gv}D_\alpha e^{gv}) \tag{2}$$

is transformed as

$$W_\alpha \to e^{-ig\Lambda}W_\alpha e^{ig\Lambda}. \tag{3}$$

In the Abelian case, the strength W_α is invariant.

Now, let us try to introduce higher derivatives in the super-Yang-Mills theory with matter (without mass or any chiral self-interaction), whose standard form is

$$S = \int d^8z\bar{\phi}e^{gv}\phi + \frac{1}{2g^2}\,\mathrm{tr}\int d^6zW^\alpha W_\alpha. \tag{4}$$

In the non-Abelian case, ϕ is a column vector (we may consider as well the case when the ϕ is not an isospinor but a Lie-algebra valued field, as W^α is, but we will limit ourselves to the isospinor case). To maintain gauge invariance one should introduce higher derivatives in a nontrivial way. For example, one of the possible forms of the action in a pure gauge sector is

$$S_W = \frac{1}{256g^2}\,\mathrm{tr}\int d^8z(e^{gv}W^\alpha e^{-gv})D^2(e^{gv}W_\alpha e^{-gv}). \tag{5}$$

However, it is not difficult to show that at the one-loop order, the self-coupling of the gauge superfield does not contribute to the Kählerian effective potential, which by definition [16] does not depend on the background gauge

*fgama@fisica.ufpb.br
†mgomes@fma.if.usp.br
‡jroberto@fisica.ufpb.br
§petrov@fisica.ufpb.br
‖ajsilva@fma.if.usp.br

superfields whose presence is unavoidable if we introduce the self-coupling of the gauge superfields in the one-loop diagrams.

So, we will restrict our study to the Abelian case where the gauge transformations are reduced to [16]

$$v \to v + i(\Lambda - \bar{\Lambda}), \tag{6}$$

and the strength W_α is explicitly gauge invariant. In this case, as the first, simplest example, we can write the following higher-derivative action:

$$\begin{aligned} S_W &= \frac{1}{256g^2}\int d^6 z W^\alpha(\Box - m^2)W_\alpha \\ &= -\frac{1}{16}\int d^8 z v D^\alpha \bar{D}^2 D_\alpha(\Box - m^2)v. \end{aligned} \tag{7}$$

We note that this action (but with $m = 0$) was earlier [8] employed as an auxiliary tool for studying a higher-derivative generalization of the Wess-Zumino model. However, detailed studies of its properties were not carried out. Notice also that in the Abelian case the action (5) reduces to

$$S_W = \frac{1}{256g^2}\int d^8 z W^\alpha D^2 W_\alpha. \tag{8}$$

Since $W_\alpha = g\bar{D}^2 D_\alpha v$, a straightforward manipulation of the above integral using the relation $\bar{D}^2 D^2 \bar{D}^2 = 16\Box\bar{D}^2$ yields

$$S_W = -\frac{1}{16}\int d^8 z v D^\alpha \bar{D}^2 D_\alpha \Box v, \tag{9}$$

which is the same as the action (7) considered henceforth, but with $m = 0$.

Let us construct now a consistent coupling to chiral matter. A natural generalization of the chiral part of the superfield action (4),

$$S_\Phi = \int d^8 z \bar{\phi} e^{gV} \mathcal{R} \phi, \tag{10}$$

would be gauge invariant only if $\mathcal{R}$ is proportional to the unit operator, both in the Abelian and in the non-Abelian case. Therefore, the simplest action for a higher-derivative supersymmetric gauge theory, that is, for the supersymmetric scalar QED, is

$$S = \int d^8 z \bar{\phi} e^{gV} \phi - \frac{1}{16}\int d^8 z v D^\alpha \bar{D}^2 D_\alpha(\Box - m^2)v. \tag{11}$$

In principle, this action may be generalized by introducing some self-couplings of the set of chiral superfields (see e.g. [17]); however, here we want to study the generic structure for the one-loop low-energy effective action of the chiral superfield.

To fix the gauge, we add the action

$$S_{gf} = \frac{1}{16\alpha}\int d^8 z v D^2 \bar{D}^2(\Box - m^2)v, \tag{12}$$

where α is the gauge-fixing parameter.

Following [16], the low-energy effective action in the theory of a chiral scalar superfield is described by the Kählerian effective potential which depends only on chiral and antichiral superfields but not on their derivatives. This effective potential will be the principal object of study in this paper. Notice that since the model (11) does not involve chiral self-coupling of the matter fields, the chiral effective potential in it will be identically equal to zero, unlike in the Wess-Zumino model and other models where such a coupling is present. The ghosts are completely factorized since the theory is Abelian.

The propagators in the theory (11) are very similar to the propagators in the usual super-Yang-Mills theory [16]:

$$\begin{aligned} \langle\phi(z_1)\bar{\phi}(z_2)\rangle &= \frac{\bar{D}^2 D^2}{16\Box}\delta^8(z_1 - z_2); \\ \langle\bar{\phi}(z_1)\phi(z_2)\rangle &= \frac{D^2 \bar{D}^2}{16\Box}\delta^8(z_1 - z_2); \\ \langle v(z_1)v(z_2)\rangle &= -\frac{1}{\Box(\Box - m^2)}\left(-\frac{D^\alpha \bar{D}^2 D_\alpha}{8\Box} + \alpha\frac{\{\bar{D}^2, D^2\}}{16\Box}\right) \\ &\quad \times \delta^8(z_1 - z_2). \end{aligned} \tag{13}$$

It is convenient to express these propagators in terms of the projection operators [16]

$$\Pi_0 = \frac{\{\bar{D}^2, D^2\}}{16\Box}, \qquad \Pi_{1/2} = -\frac{D^\alpha \bar{D}^2 D_\alpha}{8\Box}.$$

Indeed, it is clear that $\Pi_0^n = \Pi_0$, $\Pi_{1/2}^n = \Pi_{1/2}$ (for any integer $n \geq 1$), $\Pi_0\Pi_{1/2} = \Pi_{1/2}\Pi_0 = 0$. Thus, we can write

$$\begin{aligned} \langle\phi(z_1)\bar{\phi}(z_2)\rangle + \langle\bar{\phi}(z_1)\phi(z_2)\rangle &= \Pi_0\delta^8(z_1 - z_2); \\ \langle v(z_1)v(z_2)\rangle &= -\frac{1}{\Box(\Box - m^2)} \\ &\quad \times (\Pi_{1/2} + \alpha\Pi_0) \\ &\quad \times \delta^8(z_1 - z_2). \end{aligned} \tag{14}$$

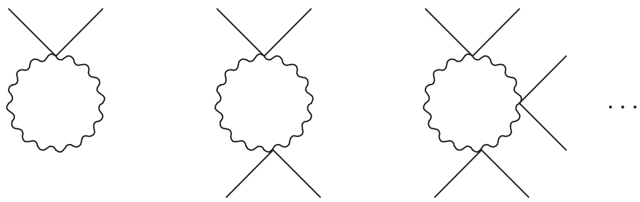
Here we emphasized the combination $\langle\phi(z_1)\bar{\phi}(z_2)\rangle + \langle\bar{\phi}(z_1)\phi(z_2)\rangle$ since it will arise in many cases, including the contributions to the one-loop Kählerian effective potential.

III. ONE-LOOP CALCULATIONS

Now, let us study the Kählerian potential. The tree approximation can be read from the classical action (4) by replacing v, W by zero and ϕ and $\bar{\phi}$ by the constant backgrounds Φ and $\bar{\Phi}$, yielding

$$K^{(0)} = \int d^8 z \Phi\bar{\Phi}. \tag{15}$$

At the one-loop order, we will have two types of contributions. In the first, all diagrams involve only the gauge field propagators in the internal lines:



The contribution of the sum of these diagrams can be expressed as

$$K_a^{(1)} = \int d^8 z_1 \sum_{n=1}^{\infty} \frac{(-1)^n}{2n} (g^2 \Phi\bar{\Phi} \frac{1}{\Box(\Box - m^2)} \times (\Pi_{1/2} + \alpha\Pi_0))^n \delta_{12}|_{\theta_1=\theta_2}, \tag{16}$$

where $\frac{1}{n}$ is a symmetry factor. These diagrams do not involve the triple vertices which will be considered shortly, only the quartic ones.

By using the properties of the projection operators, we can write

$$K_a^{(1)} = \int d^8 z_1 \sum_{n=1}^{\infty} \frac{(-1)^n}{2n} (g^2 \Phi\bar{\Phi} \frac{1}{\Box(\Box - m^2)})^n \times (\Pi_{1/2} + \alpha^n \Pi_0))\delta_{12}|_{\theta_1=\theta_2}. \tag{17}$$

Since $\frac{D^2\bar{D}^2}{16}\delta_{12} = 1$, we have $\Box\Pi_0\delta_{12}|_{\theta_1=\theta_2} = 2$ and $\Box\Pi_{1/2}\delta_{12}|_{\theta_1=\theta_2} = -2$. Thus, we have

$$K_a^{(1)} = \int d^8 z_1 \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \frac{1}{\Box} \Big(g^2 \Phi\bar{\Phi} \frac{1}{\Box(\Box - m^2)}\Big)^n \times (1 - \alpha^n)\delta^4(x_1 - x_2)|_{x_1=x_2}. \tag{18}$$

By carrying out the Fourier transform $\Box \to -k^2$ we arrive at

$$K_a^{(1)} = -\int d^8 z \int \frac{d^4 k}{(2\pi)^4} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \frac{1}{k^2} \Big(\frac{g^2\Phi\bar{\Phi}}{k^2(k^2 + m^2)}\Big)^n \times (1 - \alpha^n). \tag{19}$$

Then, by using the expansion

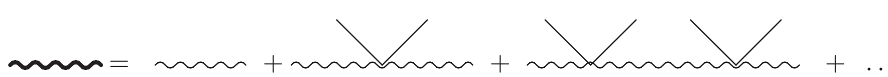
$$\sum_{n=1}^{\infty} \frac{(-a)^n}{n} = -\ln(1 + a), \tag{20}$$

we have

$$K_a^{(1)} = \int d^8 z \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} \Big[\ln\Big(1 + \frac{g^2\Phi\bar{\Phi}}{k^2(k^2 + m^2)}\Big) - \ln\Big(1 + \frac{\alpha g^2\Phi\bar{\Phi}}{k^2(k^2 + m^2)}\Big)\Big]. \tag{21}$$

Notice that at $\alpha = 0$ (Landau gauge), the second term in this expression vanishes.

The second type of diagram involves the triple vertices as well. We should first introduce a "dressed" propagator:



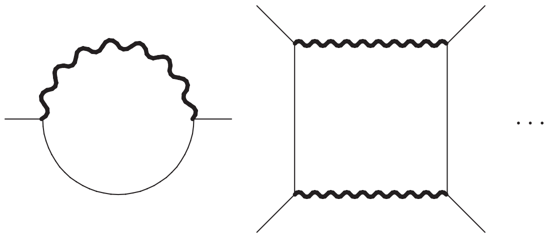
In this propagator, the summation over all quartic vertices is performed. As a result, this dressed propagator is equal to

$$\begin{aligned}\langle vv\rangle_D &= \langle vv\rangle(1 + g^2\Phi\bar{\Phi}\langle vv\rangle + (g^2\Phi\bar{\Phi}\langle vv\rangle)^2 + \ldots) \\ &= -\sum_{n=0}^{\infty} (g^2\Phi\bar{\Phi})^n \frac{1}{(\Box(\Box - m^2))^{n+1}} \times (\Pi_{1/2} + \alpha\Pi_0)^{n+1}\delta^8(z_1 - z_2).\end{aligned} \tag{22}$$

By summing up, we arrive at

$$\langle vv\rangle_D = -\Big(\frac{1}{\Box(\Box - m^2) + g^2\Phi\bar{\Phi}}\Pi_{1/2} + \frac{\alpha}{\Box(\Box - m^2) + \alpha g^2\Phi\bar{\Phi}}\Pi_0\Big)\delta^8(z_1 - z_2). \tag{23}$$

To proceed, we should sum over diagrams representing themselves as cycles of all possible numbers of links. Such diagrams look like



The complete contribution of all these cycles gives

$$K_b^{(1)} = \int d^8 z_1 \sum_{n=1}^{\infty} \frac{1}{2n} (g^2 \Phi\bar{\Phi}(\langle\phi\bar{\phi}\rangle + \langle\bar{\phi}\phi\rangle) \times \langle vv\rangle_D)^n \delta_{12}|_{\theta_1=\theta_2}, \quad (24)$$

or, similarly,

$$K_b^{(1)} = \int d^8 z_1 \sum_{n=1}^{\infty} \frac{1}{2n} (g^2 \Phi\bar{\Phi}\Pi_0\langle vv\rangle_D)^n \delta_{12}|_{\theta_1=\theta_2}. \quad (25)$$

By noting that

$$\Pi_0\langle vv\rangle_D = -\frac{\alpha}{\Box(\Box - m^2) + \alpha g^2\Phi\bar{\Phi}} \Pi_0, \quad (26)$$

we can rewrite the expression above as

$$K_b^{(1)} = \int d^8 z_1 \sum_{n=1}^{\infty} \frac{(-1)^n}{2n} \times \left(\frac{\alpha g^2\Phi\bar{\Phi}}{\Box(\Box - m^2) + g^2\Phi\bar{\Phi}}\right)^n \Pi_0 \delta_{12}|_{\theta_1=\theta_2}. \quad (27)$$

Since $\Box\Pi_0\delta_{12}|_{\theta_1=\theta_2} = 2$, we have

$$K_b^{(1)} = \int d^8 z_1 \sum_{n=1}^{\infty} \frac{1}{n}\frac{1}{\Box}\left(-\frac{\alpha g^2\Phi\bar{\Phi}}{\Box(\Box - m^2) + g^2\Phi\bar{\Phi}}\right)^n \times \delta^4(x_1 - x_2)|_{x_1=x_2}. \quad (28)$$

By carrying out the Fourier transform and the summation as above, we arrive at

$$K_b^{(1)} = \int d^8 z \int \frac{d^4k}{(2\pi)^4} \frac{1}{k_E^2}\left[\ln\left(1 + \frac{\alpha g^2\Phi\bar{\Phi}}{k_E^2(k_E^2 + m^2)}\right)\right]. \quad (29)$$

By summing this contribution to $K_a^{(1)}$ (21), we see that the α-dependent contribution vanishes, and the total one-loop Kählerian effective potential is gauge independent, being equal to

$$K^{(1)} = \int d^8 z \int \frac{d^4k}{(2\pi)^4} \frac{1}{k_E^2} \ln\left[1 + \frac{g^2\Phi\bar{\Phi}}{k_E^2(k_E^2 + m^2)}\right]. \quad (30)$$

This integral, after the changes $k^2 = m^2 u$ and $d^4k = \pi^2 m^4 u du$, can be rewritten in spherical coordinates,

$$K^{(1)} = m^2 \int d^8 z \int_0^{\infty} \frac{du}{(4\pi)^2} \ln\left[1 + \frac{g^2\Phi\bar{\Phi}/m^4}{u(u+1)}\right]. \quad (31)$$

The computation of this integral is straightforward and furnishes

$$K^{(1)} = -\frac{m^2}{16\pi^2} \int d^8 z[x_+ \ln(x_+) + x_- \ln(x_-)], \quad (32)$$

where $x_\pm = [1 \pm (1 - 4g^2\Phi\bar{\Phi}/m^4)^{1/2}]/2$. Thus

$$K^{(1)} = -\frac{m^2}{16\pi^2} \int d^8 z\left[\ln\left(\frac{g^2\Phi\bar{\Phi}}{m^4}\right) + \frac{1}{2}\left(1 - 4\frac{g^2\Phi\bar{\Phi}}{m^4}\right)^{1/2} \ln\left(\frac{x_+}{x_-}\right)\right] \quad \text{for} \quad \frac{4g^2\Phi\bar{\Phi}}{m^4} < 1, \quad (33)$$

and

$$K^{(1)} = -\frac{m^2}{16\pi^2} \int d^8 z\left[\ln\left(\frac{g^2\Phi\bar{\Phi}}{m^4}\right) - \left(\frac{4g^2\Phi\bar{\Phi}}{m^4} - 1\right)^{1/2} \arctan\left(\frac{4g^2\Phi\bar{\Phi}}{m^4} - 1\right)^{1/2}\right] \quad (34)$$

for $\frac{4g^2\Phi\bar{\Phi}}{m^4} > 1$. In particular, for $m \to 0$ we find

$$K^{(1)} = \frac{1}{16\pi} \int d^8 z (g^2\Phi\bar{\Phi})^{1/2}. \quad (35)$$

It is worth noticing that our result is finite and does not need any renormalization. In other words, the one-loop Kählerian effective potential to the higher-derivative supersymmetric Abelian gauge theory does not display any divergences, unlike the usual gauge theories [11].

IV. SUMMARY

We have explicitly found the one-loop Kählerian effective potential in the supersymmetric higher-derivative QED. We note that the chiral contributions to the effective action, which are known to be typical for the Wess-Zumino model and its straightforward many-field generalizations [12], simply do not arise due to the absence of the chiral self-interaction in the classical action. However, had we introduced such a term, it is most likely that the chiral contributions to the effective action could either display infrared singularities due to the augmented degree of momenta in the denominator (compare with [18] for the usual supersymmetric QED), in the case where the higher-derivative theory is massless, or give a zero result in the opposite case (one should note that, in the Wess-Zumino model, the chiral effective potential does not emerge in the massive case either).

We have restricted our study to the Euclidean region to evade known difficulties in the formulation of higher-derivative theories in Minkowski space [19]. As discussed in [15], the extension of the results to Minkowski space would probably demand the inclusion of additional interactions to compensate the singularities met in the process of analytic continuation. The natural continuation of this work could consist in the formulation and study of more generic higher-derivative supersymmetric gauge models. We will return to this problem in a forthcoming paper.

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